Measurements and Uncertainties

Sources of errors :-

1- Choice of instruments
2- The Expirementer
3- The Environment
4- The way The expirement is done
5- The way The physical quantity is measured

- A measurement can never be take without any error. But it can be estimated when all errors are very small

errors → Random errors
- uncertainty in a measurement (we estimate it)
- ~ ~ ~ several ~ (The mean)

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i \quad : \text{best estimation}$$

$$\sigma_s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \quad : \text{Sample standard deviation}$$

$$\sigma_m = \frac{\sigma_s}{\sqrt{N}} \quad : \text{standard deviation of The mean}$$

errors → systematic errors
- خطأ في الغرض looking perpendicularly not in a wrong way
- عدم البدء من التدريج الصفري
- الجهاز غير معاير uncalibrated Instrument

Precision and Accuracy

Random errors
- High → less precision
- low → more precision

Exp: 9.82 ± 0.5 less
9.8 ± 0.1 More precise

Systematic error
- High → less Accuracy
- low → More Accuracy

Exp: True value: 9.86
$X_A = 9.8 \pm 0.01$ → More Accurate
$X_B = 10.1 \pm 0.4$ → less Accurate

Discrepancy test : accepted / not accepted

* True value X
* Result $\bar{X} \pm \Delta X$

- steps:

1- $D = |X - \bar{X}|$

2- $2^x \Delta X$

3- if $D > 2\Delta X$ not accepted

if $D \leq 2\Delta X$ accepted

significant figures

• هي الأرقام المعنوية التي يعبر عنها

Exp: 900: 1 significant
900.: 3 ~
900.0: 4 sig
0.020: 2 sig

ΔX: should always be 1 sig figure unless the leading digit was one Then we keep the digit after Exp 0.123 ≈ 0.12
0.16 ≈ 0.2 or 1.6

Rounding Rules :-

• any number less than 5 ← we fix the sig. fig
• ~ ~ more ~ 5 we round the last sig fig up
• If it was 5. Exp 3.5 → 40
رقم الفردي

45 → 40
رقم الزوجي

0.7251 → 0.73
رقم فردي

Values

* Addition and substraction

- The n° with the fewest decimal places limits the number of decimal places in the result

الناتج العشرية

* Multiplication and division

- we find how much of sig fig ther is in the numbers Multiplied : The less controls The result

- $\sqrt{15} = 3.782 \approx 3.8$ (2 s.f)

$\sqrt{2.4+10.2} = \sqrt{12.6} = 3.549$ تصبح قلة الجمع

- $\sin(24) = (0.406) \approx 0.41$
- $\cos(70) = 0.342 \approx 0.3$

Uncertainity

* Addition and substraction

$R = X \pm y \quad \Delta R = \Delta x + \Delta y$: general rule

* Constant Multipliers

$R = ax \pm by \quad \Delta R = a\Delta x + b\Delta y$

But if a and b are not const Then

$\Delta R = a\Delta x + x\Delta a + b\Delta y + \Delta b$

* Multiplication and division

$A = xy$

$\frac{\Delta A}{A} = \frac{y\Delta x + x\Delta y}{A}$ for 2 values

$\frac{\Delta A}{A} = \frac{y\Delta x}{xy} + \frac{x\Delta y}{xy}$ for more Than 2 values

$= \frac{\Delta x}{x} + \frac{\Delta y}{y}$

* Raising to powers

$$R = X^l y^m Z^h$$

$$\frac{\Delta R}{R} = |l| \frac{\Delta x}{X} + |m| \frac{\Delta y}{y} + |h| \frac{\Delta z}{z}$$

* $R = e^x$ (العدد النيبي)

↳ $\Delta R = e^x \Delta x$

* $R = \ln X$

↳ $\Delta R = \frac{1}{X} \Delta x$

* $R = \sin \theta$

↳ $\Delta R = \cos\theta \, \Delta\theta$

* $R = \cos\theta$

↳ $\Delta R = |-1| \sin\theta \, \Delta\theta$

لـ Rad ↲ تحول الخطأ صار الزوايا

عن طريق $2\pi \rightarrow 180$

→ الاشتقاق الجزئي

• $R = R(X, y, z)$

↳ • $\Delta R = \left|\frac{\partial R}{\partial x}\right| \Delta x + \left|\frac{\partial R}{\partial y}\right| \Delta y + \left|\frac{\partial R}{\partial z}\right| \Delta z$

اشتقاق جزئي

Experiment 1

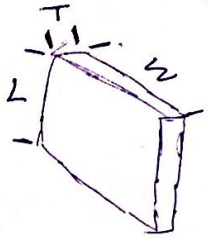
to identify the material

- Density & Distance between Atoms

Theory:

Density = $\rho = \frac{M}{V} = L \times W \times T$

M: Mass, V: Volume



- In Metal: atoms are spherical & identical
  = (lattice structure) almost

Total number of atoms:

$$N = (n) N_a = \frac{M}{Aw} N_A$$

M: Mass, Aw: The Atomic mass of The Material, N_A: Avogadro's no.

or

$$N = \frac{M}{\rho a^3} \Rightarrow a = \sqrt[3]{\frac{Aw}{\rho N_a}}$$

now uncertainty in ρ:

$$\Delta\rho = \frac{\Delta M}{V} + \frac{M}{V^2}\Delta V$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$$

ΔM: estimated

$\Delta V = WT\Delta L + \Delta WTL + W\Delta TL$

$$\frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta W}{W} + \frac{\Delta T}{T}$$

Exp#2: Conservation of linear Momentum

$\vec{P} = m\vec{v}$ → velocity, m → Mass

$\vec{P}$ ← linear Momentum

- if there was N objects in an isolated system:- → No External resultant forces acts on it

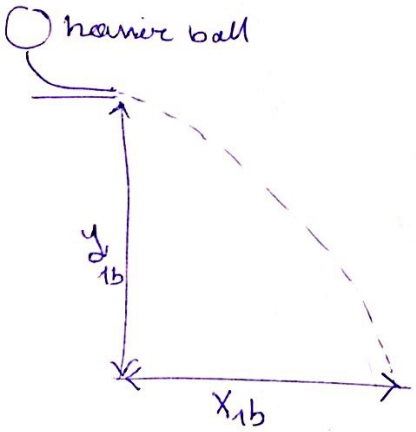
$$\vec{P} = \sum_{i=1}^{N} m_i \vec{v_i}$$

$\vec{P}$: is conserved for an isolated system

* Collision:-

$\vec{P}$ before collision = $\vec{P}$ after collision

$$m_1 v_{1b} + \underbrace{m_2 v_{2b}}_{=0} = m_1 v_{1a} + m_2 v_{2a}$$



Theory. $\frac{P_a}{P_b} = \frac{m_1 v_{1a} + m_2 v_{2a}}{m_1 v_{1b}} = 1$

$y_{1b} = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2y}{g}}$

$v_b = \frac{x_b}{t_b} \Rightarrow V = \frac{x}{\sqrt{2yg}}$

$P_b = \frac{m x_b}{\sqrt{\frac{2y}{g}}}$ • it is equal for the so 2 balls before and after

$\frac{P_a}{P_b} = R = \frac{m_1 x_1 a + m_2 x_2 a}{m_1 x_{1b}} = \frac{A}{B}$

$\frac{\Delta R}{R} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{m_1 \Delta x_a + x_{1a} \Delta m_1 + m_2 \Delta x_{2a} + x_{2a} \Delta m_2}{m_1 x_1 a + m_2 x_2 a} + \frac{m_1 \Delta x_b + x_b \Delta m_1}{m_1 x_{1b}}$

Exp 3: Density of liquids

Fluids → gases, liquids

- exert forces on the walls of their containes (perpicular to the surface)

pressure

- $P = \frac{F}{A}$ → force, → area
- pressure is larger at lower points
- a portion of liquid :

$$P_2 A - mg - P_1 A = 0$$

$$A(P_2 - P_1) = mg$$

$$(P_2 - P_1) = \frac{mg}{A}$$

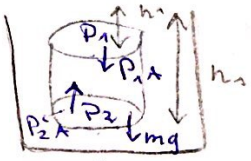
$$P = \frac{m}{V}$$

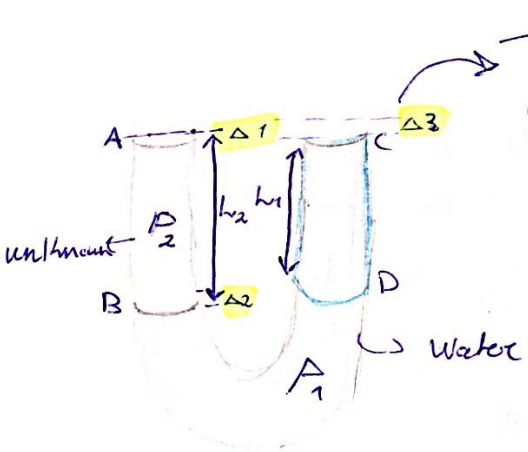
$$m = \rho V$$

$$P_2 - P_1 = \frac{\rho V g}{A}$$

$$P_2 - P_1 = \frac{\rho A (h_2 - h_1) g}{A}$$

$$P_2 - P_1 = (h_2 - h_1)\rho g$$





- U-Tube :-

1- $P_B - P_A = L_2 \times \rho_2 \times g$

2- $P_D - P_C = h_1 \times \rho_1 \times g$ { B=D, A=C

$$L_2 \rho_2 g = L_1 \rho_1 g$$

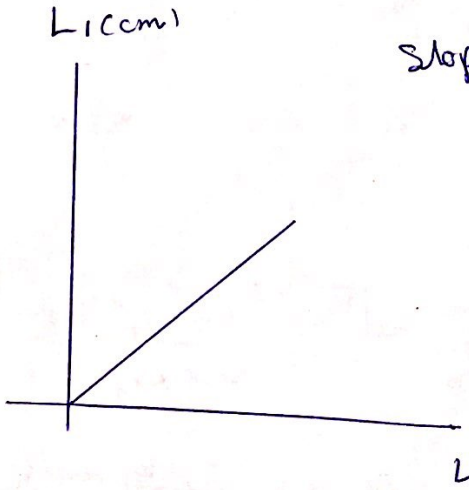
$$L_2 \rho_2 g = L_1 g$$

$$\rho_2 = \frac{L_1}{L_2}$$

$$\frac{\Delta P}{\rho} = \frac{\Delta L_1}{L_1} + \frac{\Delta L_2}{L_2}$$

$\Delta 1 + \Delta 2$

$\Delta 2 + \Delta 3$



$$\text{Slope} = \frac{\Delta L_1}{\Delta L_2} = \rho$$

Exp 4 :- Dc circuits

- $R = \frac{\text{Voltage}}{\text{Current}} = \frac{V}{I}$ = فرق الجهد Potential difference / شدة التيار Current flowing

Resistance of a metalic conductor

Materials:
- ohmic : V depends linearly on I
- non-ohmic : V does not depend linearly on I

Equivalent Resistance of 2 Resistors

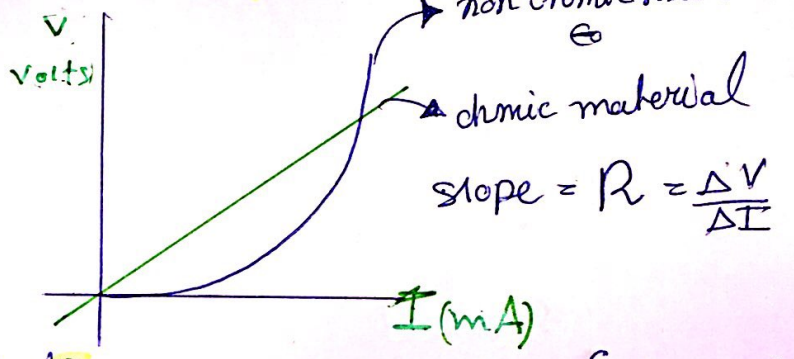
I is the same for R_1 & R_2

$R_{serie} = R_1 + R_2$

$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$

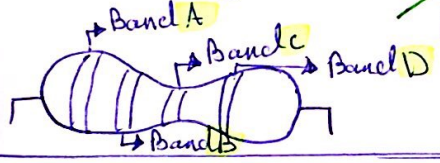
V is the same for R_1 & R_2

$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$



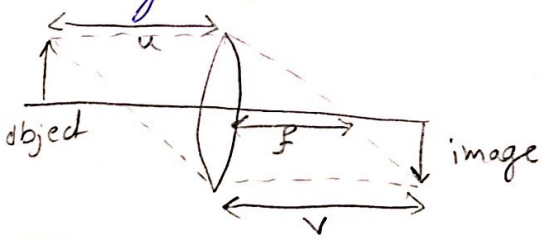
slope $= R = \frac{\Delta V}{\Delta I}$

Color code



R (Theoritically) $= AB \times 10^{C} \pm (D \times *R)$

Exp 5 :- focal length of a Convex lens



Focal length:
- The distance between the lens and the point of convergence of the light rays coming from the infinity

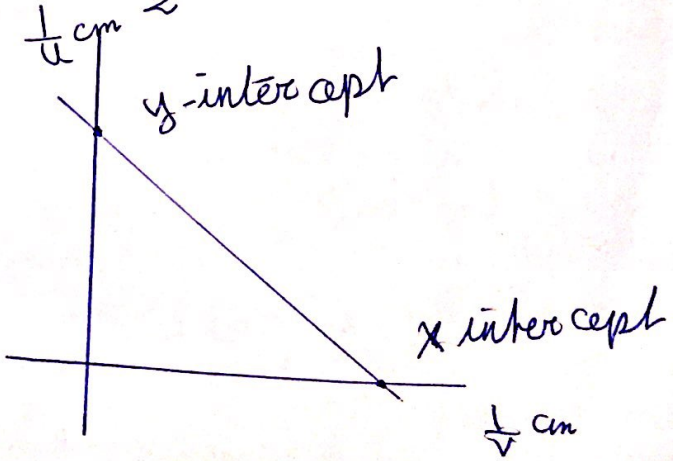
Focal length $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ ($\frac{1}{u}$: y-intercept, $\frac{1}{v}$: x-intercept)

$\Rightarrow \overline{f} = \frac{f_x + f_y}{2}$

$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$

$f_x = f_y$ (Theoritically)

$\frac{1}{\text{x-intercept}} = \frac{1}{\text{y-intercept}}$



Exp5 :-

- If the object is placed At infinity Then the image will be formed at f

Exp 4

- Voltmeter on parallel:

is a device used to measure The potential difference and it has a high ~~Resista~~ Resistance and if we connect

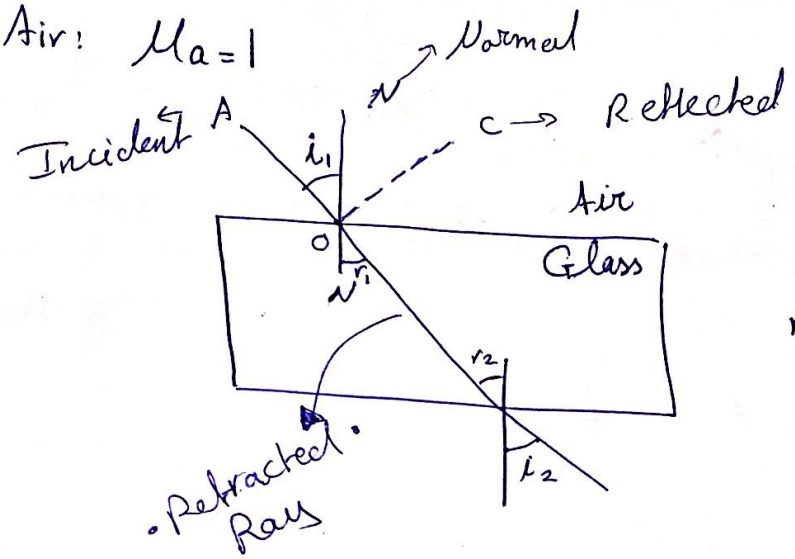
It on serie Then it will impede the current and no Reading will show
and the Ammeter, If we connect it on parallel (It has a low resistance) a big amount of current would go through one branch and the Ammeter will burn out

EXP 6 - Index of Refraction

$n = \frac{c}{v}$ → speed of light in vacuum
→ speed of light in medium

- The light bends when moving from a medium to another

for Air: $\mu_a = 1$



i: angle of incidence
r: angle of Refraction

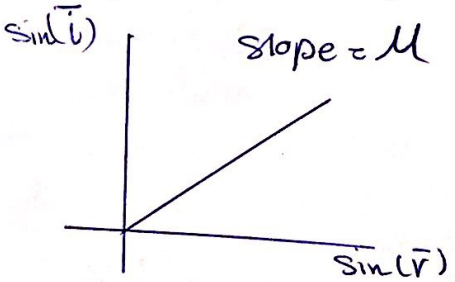
Snell's law: $\mu_a \sin(i) = \mu_g \sin(r)$
(Angle of incidence ↵, ↳ angle of Refraction)

$\mu_a = 1$

$$\sin(i) = \mu_g \sin(r)$$

$$\mu_g = \frac{\sin(i)}{\sin(r)}$$

- μ_g is the slope



$$\frac{\Delta \mu_g}{\mu_g} = \frac{\Delta \sin(i)}{\sin i} + \Delta \frac{\sin(r)}{\sin r}$$

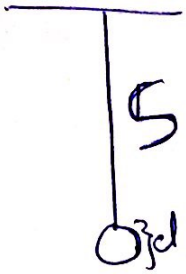
$$\frac{\Delta \mu_g}{\mu_g} = \frac{\cos i}{\sin i}\Delta i + \frac{\cos r}{\sin r}\Delta r$$

Δi and Δr in radians

↓

Δ estimation

Exp 7 Measuring g at BZU using least square fit method



$$L = S + \frac{d}{2}$$

T (period) = time required by the pendulum to finish one oscillation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

• only when θ is small < 15

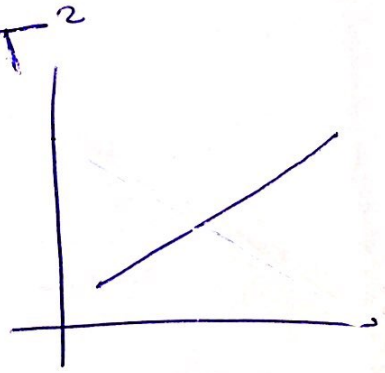
$$T^2 = 4\pi^2\left(\frac{l}{g}\right)^{1*}$$

T^2 = y, $4\pi^2$ = m, $\left(\frac{l}{g}\right)^{1*}$

$$m = (\text{best slope}) = \frac{4\pi^2}{g}$$

$$\frac{\Delta g}{g} = \frac{\Delta m}{m}$$

• finding the value of the slope m, the y-intercept b and their uncertainties using the least square fit method



best slope $= \frac{4\pi^2}{g}$

$m = \frac{4\pi^2}{g}$

$g = \frac{4\pi^2}{m}$

Exp 8: half-life of a draining water Column

$$\frac{dh}{dt} = -\lambda\, h(t)$$

$$\Rightarrow \int_{h_0}^{h(t)} \frac{dh}{h} = \int_0^t -\lambda\, dt$$

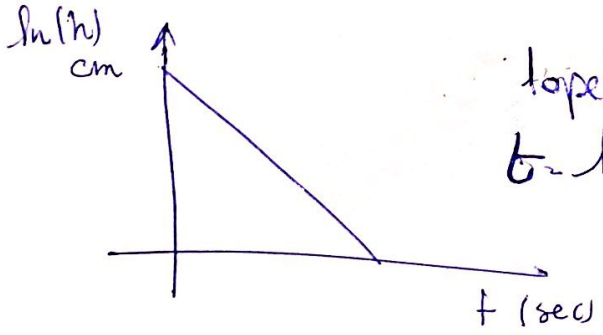
$h_0 = \underbrace{50 \text{ units} + D}_{\text{in burette units}}$

$$\boxed{h(t) = h_0\, e^{-\lambda t}}$

~~best slope~~

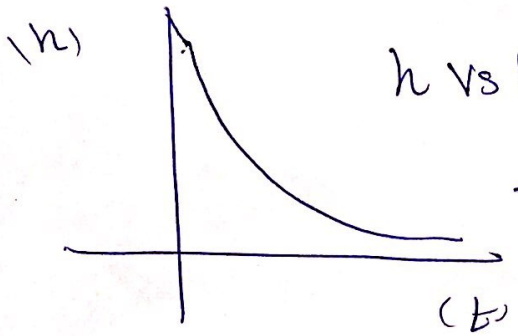
$\Rightarrow$ when $t_{1/2} \Rightarrow \frac{h_0}{2} = h_0\, e^{-\lambda t_{1/2}}$

$$\Rightarrow \boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$



slope $= -\lambda$

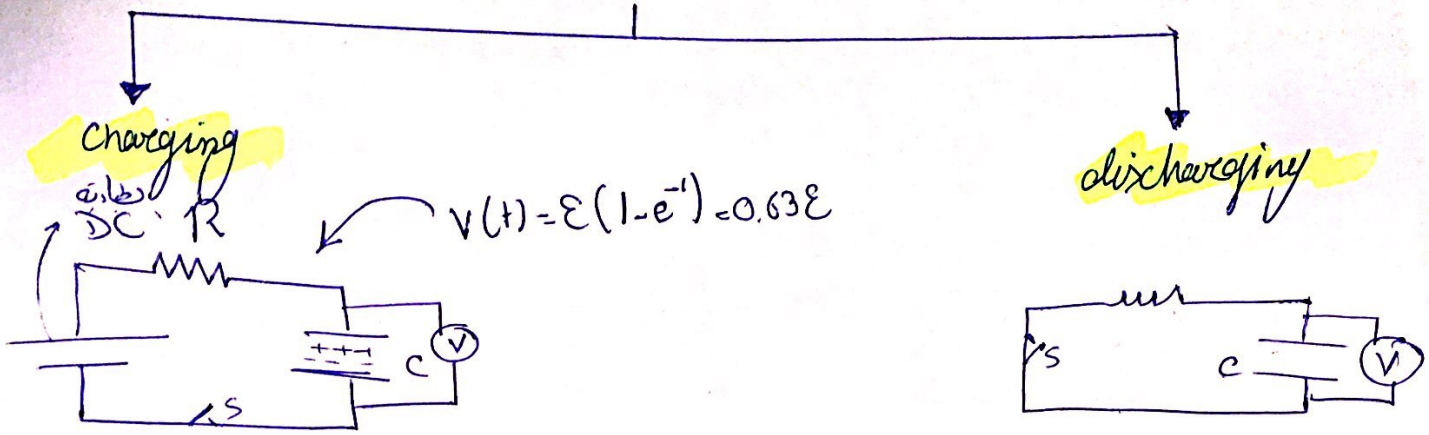
$t_{1/2} = \frac{\ln 2}{\lambda}$



h vs t

→ to obtain 6 measurements of $t_{1/2}$

Exp 9 : RC circuit



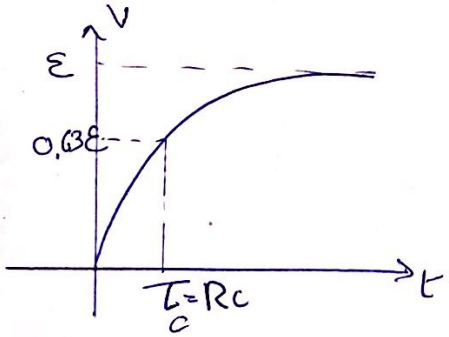
$V = \frac{Q}{C}$ فولت

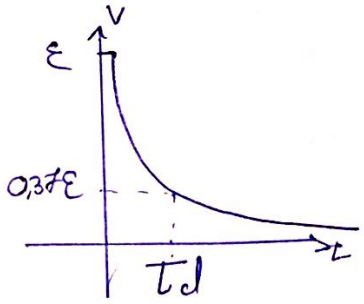
$[C] = \text{Farad} = \frac{\text{Colum}}{\text{Volts}}$

$V_C = 0.37\mathcal{E}$

$\tau = RC$

$V_C = 0.63\mathcal{E}$

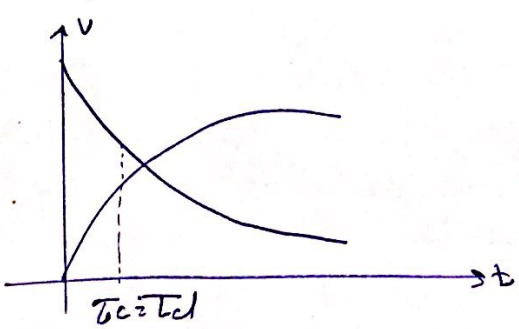




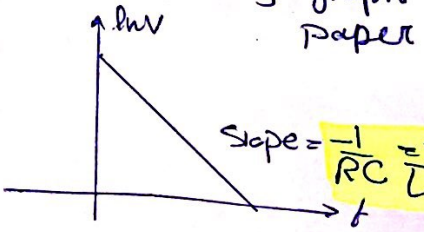
$\tau_c = \tau_d$ نظرياً

$\bar{\tau} = \frac{\tau_c + \tau_d + \tau_s}{3}$ عملياً

Theoritically



- on a semi-log graph paper



$\Rightarrow C = \frac{\bar{\tau}}{R}$

$\frac{\Delta C}{C} = \frac{\Delta \bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R}$

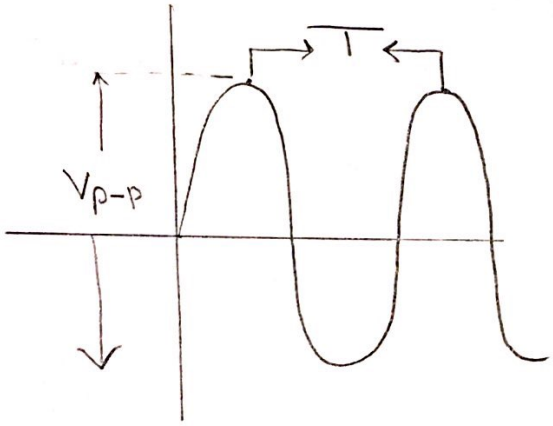
where $\Delta\bar{\tau}$ is calculated = σ_m

The Cathode-Ray Oscilloscope (CRO)

used for 1- measuring the peak to peak voltage V_{p-p}

2- measuring the frequency of a sinusoidal signal supplied by a signal generator

3- To display lissajous figures

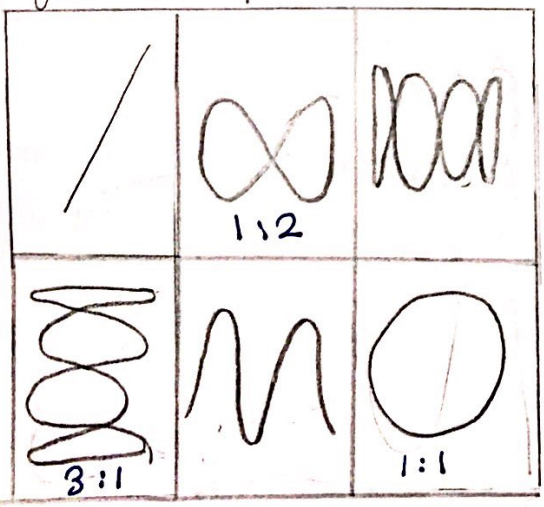


$f = \frac{1}{T}$ (frequency)

$V_p = \frac{V_{p-p}}{2}$

lissajous figures

for example



- formes of waves
  - Sine Wave
  - Square Wave
  - Triangle Wave
  - Saw tooth

- CRO can display graphs of potential differences Vs. time
- it can be used to measure AC and DC voltages
- Can misure amplitude & freequency of a given AC signal As well as the phase (Φ) betwee two AC Signals

Structure

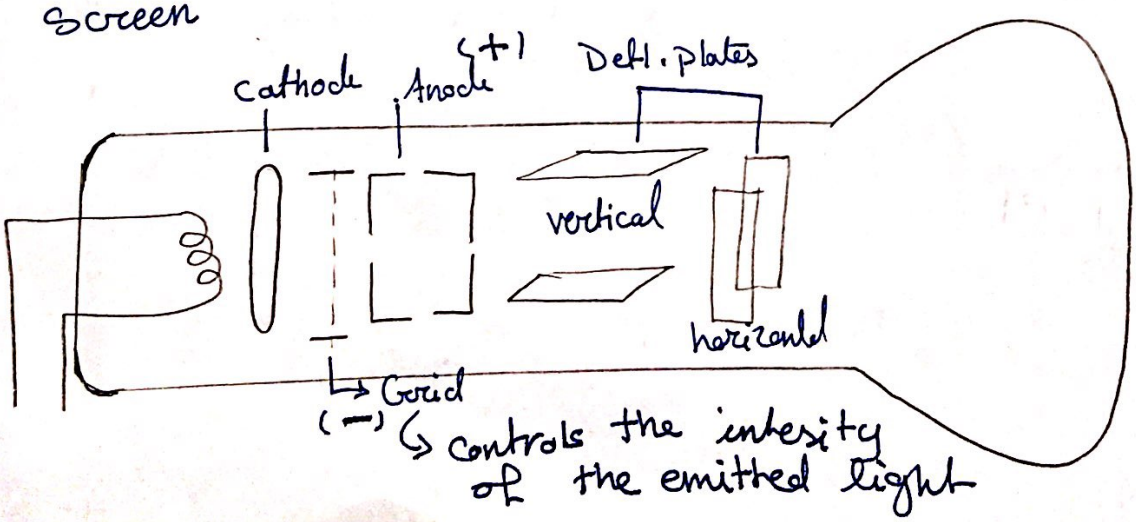
cathode Ray Tube

1- evacuated glass tube (CRT)
2- Cathode and Anode
3- Deflection plates
4- Grid (charged -)
5- vertical and horizontal deftection Plates

The process :-

① filaments heates the cathode ⇒ ② electrons are emitted by the Cathode ⇒ ③ electrons are accelerated due the high Positive potential at the accelerating anode

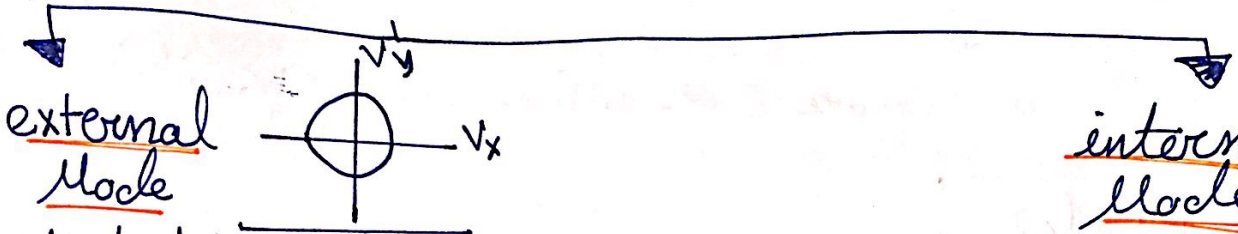
⇒ ④ electrons move toward the florescent screen ⇒ ⑤ electrons hit the screen ⇒ ⑥ The material covering the screen emits light



To Summerize the process :- 6 steps

Cathode is heated → electrons are emitted → electrons are accelerated → electrons move toward the screen → electrons hit the screen → screen emits light

Modes of the CRO

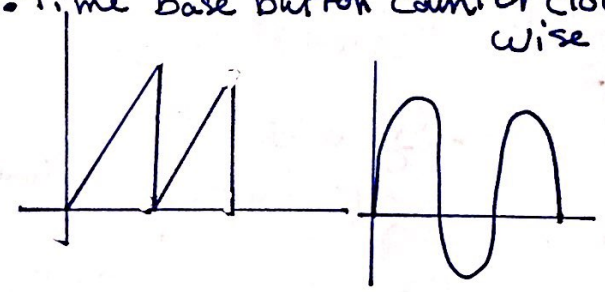


external Mode

- selected by the time base button to the x-y ext. mode
- screen acts as an x-y plotter
- The voltage .Vs Time plot appears on the screen
- lissajous figures
- Time base button clockwise

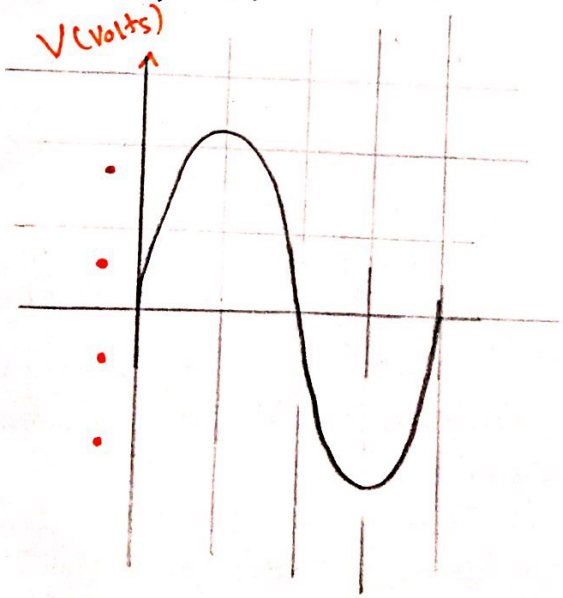
internal Mode

- x-axis becomes a time axis
- a sawtooth ~~wave~~ potential difference
- Time base button counter clock wise



- x- input :- recieves external signals
- y- " :- " " "

T (period) = No of boxes of one wave x time base Reading



$V_{p-p} = 4 \times 1$ Volts

$= 4$ Volts

$T = 4 \times 1$ se

$= 4$ sec

In case That Time base Reading is

1 Volts / square

1 sec / square

Expll) RC Circuit using Oscilloscope

- measuring τ and $t_{1/2}$ using a signal generater and and a CRO

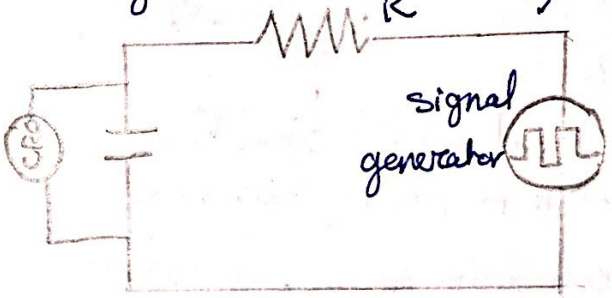
- Connecting a signal generater and R and C on sereies
- (provides a square Wave voltage to the circuit)

$$t_{\frac{1}{2}} = RC \ln 2$$

$$t_{\frac{1}{2}} = \tau \ln 2$$

$$V(t) = \frac{Q_0}{C}(1 - e^{\frac{t}{RC}}) \quad \text{(charging)} \qquad \tau_c = 0.63 V_{max}$$

$$V(t) = \frac{Q_0}{C} e^{\frac{t}{RC}} \quad \text{(discharging)} \qquad \tau_d = 0.37 V_{max}$$



-: CRO على الـ نظهر